Performance Analysis of IIR Filter Design by Using Butterworth, Chebyshev and Cauer

Mandeep Singh Saini[1], Dr. Kuldeep Kaur[2]

[1]M.Tech. (E.C.E), from Guru Nanak Dev Engineering College, Ludhiana .P.T.U Regional Centre, (Punjab) India.
[2]Ph. D (Asst. Prof.), Faculty Member of B.A.M. Khalsa College, Garhshankar. (Punjab)

Abstract *- In this paper we examine the performance of IIR filters. Cauer or Elliptical IIR filter cost performance is compare with other Butterworth filter and Chebyshev filter. We have been analyzed on the basis of filter order, multiplier, adder, no of states, MPIS and APIS. The Elliptical IIR Filter is examined and comparison from the Butterworth and Chebyshev is done. The Elliptical Filter is minimizing the order of filter optimal to 75% to Butterworth Filter, and 42.5% to Chebyshev.*

Keywords:-Butterworth, Chebyshev, Cauer, IIR, FIR

[A] INTRODUCTION

1. Need of IIR Filters

Digital filters with finite-duration impulse response (all-zero, or FIR filters) have both advantages and disadvantages compared to infinite-duration impulse response (IIR) filters.

FIR filters have the following primary advantages:

- They can have exactly linear phase.
- They are always stable.
- The design methods are generally linear.
- They can be realized efficiently in hardware.
- The filter startup transients have finite duration.

The primary disadvantage of FIR filters is that they often require a much higher filter order than IIR filters to achieve a given level of performance. Correspondingly, the delay of these filters is often much greater than for an equal performance IIR filters.

The IIR Filter parameter allows you to specify Butterworth, Chebyshev type I, Chebyshev type II, and elliptic filter designs. Note that for the band pass and band stop configurations, the actual filter length is twice the Filter order parameter value. [7]

TABLE 1.1
IIR FILTER TYPES

Filter Design	Description
Butterworth	The magnitude response of a Butterworth filter is maximally flat (i.e. has no Ripples) in the pass band and monotonic overall.
Chebyshev type I	The magnitude response of a Chebyshev type I filter is equiripple in the pass band and monotonic in the stop band.
Chebyshev type II	The magnitude response of a Chebyshev type II filter is monotonic in the pass band and equiripple in the stop band.
Elliptic	The magnitude response of an elliptic filter is equiripple in both the pass band and the stop band.

2. BUTTERWORTH FILTER

The gain of an *n*-order Butterworth low pass filter is given in terms of the transfer function *H(s)* as [6].

$$G^2(\omega) = |H(j\omega)|^2 = \frac{{G_0}^2}{1+\left(\frac{\omega}{\omega_C}\right)^{2n}} \quad \ldots\ldots\ldots. \quad 2.1$$

Where

- n = order of filter
- ω_c= cutoff frequency (approximately the -3dB frequency)
- G_0 is the DC gain (gain at zero frequency)

It can be seen that as *n* approaches infinity, the gain becomes a rectangle function and frequencies below ω_c will be passed with gain G_0, while frequencies above ω_c will be suppressed. For smaller values of *n*, the cutoff will be less sharp.

We wish to determine the transfer function *H(s)* where $S = \sigma + j\omega$ (from Laplace transform).

Since $|H(S)|^2 = H(S)\overline{H(S)}$ and as a general property of Laplace transforms at $S = j\omega$ $H(-j\omega) = \overline{H(-j\omega)}$ [6]. , then if we select H(s) such that:

$$H(S)H(-S) = \frac{{G_0}^2}{1+\left(\frac{-s^2}{\omega_c^2}\right)^n} \quad \ldots\ldots\ldots\ldots\ldots\ldots. \quad 2.2$$

Then for imaginary inputs, $S = j\omega$, we have the frequency response of the Butterworth filter.

The n poles of this expression occur on a circle of radius ω_c at equally-spaced points, and symmetric around the imaginary axis. For stability, the transfer function, H(s), is therefore chosen such that it contains only the poles in the negative real half-plane of *s*. The k^{th} pole is specified by

$$-\frac{s_k^2}{\omega_c^2} = (-1)^{\frac{1}{n}} = e^{-\frac{j(2\pi-1)\pi}{n}} \qquad k = 1,2,3 \ldots\ldots$$

And hence

$$S_K = \omega_C\, e^{-\frac{j(2\pi-1)\pi}{n}} \qquad k = 1,2,3, \ldots\ldots\ldots$$

The transfer function may be written in terms of these poles as [6]

$$H(S) = \frac{G_0}{\prod_{k=1}^{n} {(s-s_k)}/{\omega_c}} \quad \ldots\ldots\ldots. \quad 2.3$$

The denominator is a Butterworth polynomial in *s*.

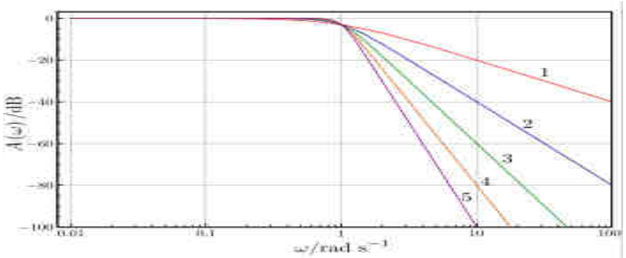


Fig. 2.1 Plot of the gain of Butterworth low-pass filters. [6]

Where G_0 is the filter gain and $\omega_C = 0.11453$ is the 3dB cut-off frequency and N=18 is the order of the filter. The magnitude response of the Butterworth filter is shown in figure 2.1.

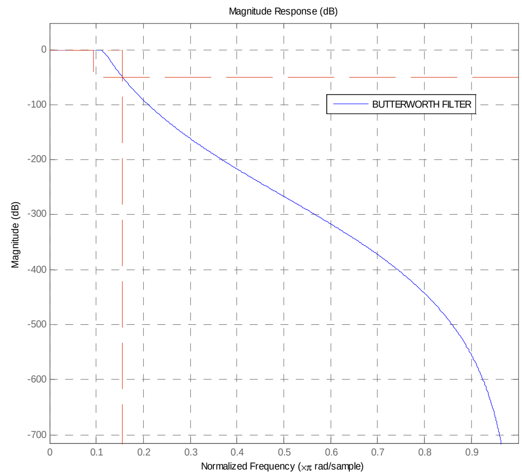
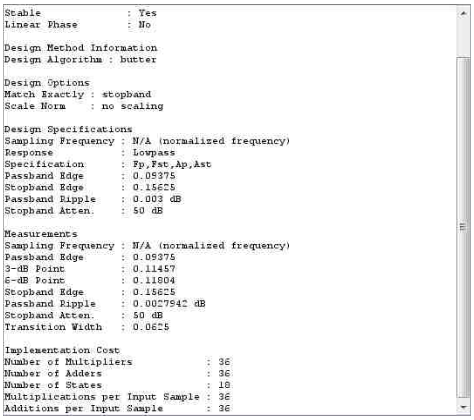


Fig. 2.2 Magnitude response of a butterworth Low pass fil



```
Stable                : Yes
Linear Phase          : No

Design Method Information
Design Algorithm : butter

Design Options
Match Exactly : stopband
Scale Norm    : no scaling

Design Specifications
Sampling Frequency : N/A (normalized frequency)
Response           : Lowpass
Specification      : Fp,Fst,Ap,Ast
Passband Edge      : 0.09375
Stopband Edge      : 0.15625
Passband Ripple    : 0.003 dB
Stopband Atten.    : 50 dB

Measurements
Sampling Frequency : N/A (normalized frequency)
Passband Edge      : 0.09375
3-dB Point         : 0.11457
6-dB Point         : 0.11804
Stopband Edge      : 0.15625
Passband Ripple    : 0.0027942 dB
Stopband Atten.    : 50 dB
Transition Width   : 0.0625

Implementation Cost
Number of Multipliers             : 36
Number of Adders                  : 36
Number of States                  : 18
Multiplications per Input Sample  : 36
Additions per Input Sample        : 36
```

FILTER ORDER	MUL.	ADDER	STATES	MPIS	APIS
18	**36**	**36**	**18**	**36**	**36**

Fig. 2.3 cost of a Butterworth Low pass filter

3. Chebyshev type I

Chebyshev filters are analog or digital filters having a steeper roll-off and more pass band ripple (type I) or stop band ripple (type II) than Butterworth filters.

Type I Chebyshev filters are the most common types of Chebyshev filters. The gain (or amplitude) response as a function of angular frequency ω of the *n*th-order low-pass filter is equal to the absolute value of the transfer function $H_n(j\omega)$

$$G_n(\omega) = |H_n(j\omega)| = \frac{1}{\sqrt{1+\varepsilon^2\left(\frac{\omega}{\omega_0}\right)T_n^2}} \quad \ldots \ldots \qquad 3.1$$

Where ϵ is the ripple factor, ω_0 is the cut off frequency and T_n is a Chebyshev polynomial of the n^{th} order.

The pass band exhibits equiripple behavior, with the ripple determined by the ripple factor ϵ. In the pass band, the Chebyshev polynomial alternates between -1 and 1 so the filter gain alternate between maxima at $G = 1$ and minima at $G = {}^{1}/_{\sqrt{1+\varepsilon^2}}$. At the cutoff frequency ω_0 the gain again has the value ${}^{1}/_{\sqrt{1+\varepsilon^2}}$ but continues to drop into the stop band as the frequency increases. This behavior is shown in the diagram on the right. The common practice of defining the cutoff frequency at −3 dB is usually not applied to Chebyshev filters; instead the cutoff is taken as the point at which the gain falls to the value of the ripple for the final time.

The order of a Chebyshev filter is equal to the number of reactive components (for example, inductors) needed to realize the filter using analog electronics.

The ripple is often given in dB:

$$\text{Ripple in dB} = \ 20\ \log_{10}\sqrt{1+\varepsilon^2}$$

So that a ripple amplitude of 3 dB results from $\varepsilon = 1$.

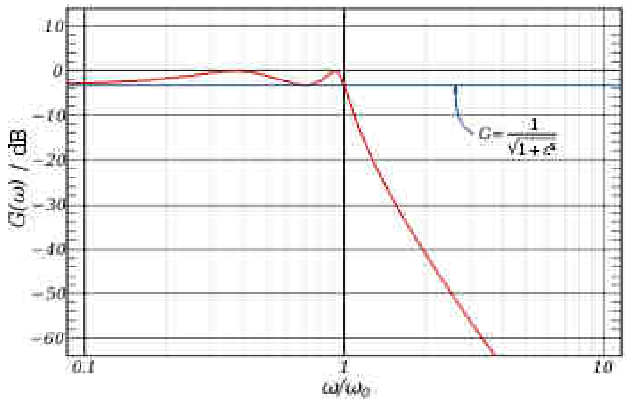


Fig.3.1 The frequency response of a 4th order type 1 chebyshev low pass filter with $\varepsilon = 1$ Error! Reference source not found. [6]

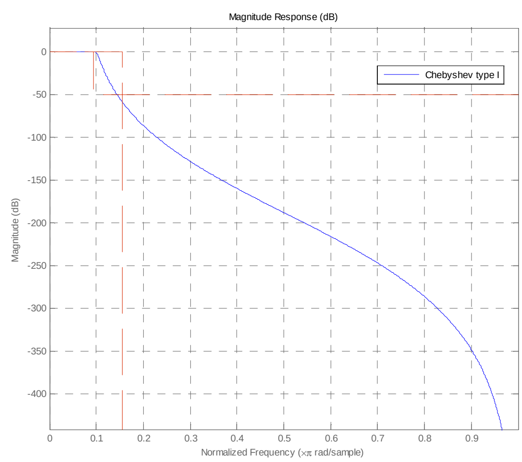
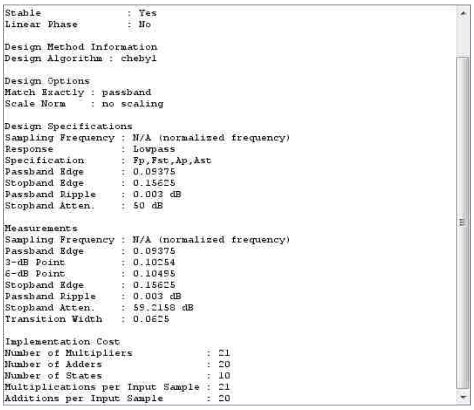


Fig. 3.2 Magnitude response of a Chebyshev-1 Low pass filter with $\varepsilon = 0.02628148$



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Stable                : Yes
Linear Phase          : No

Design Method Information
Design Algorithm : cheby1

Design Options
Match Exactly : passband
Scale Norm    : no scaling

Design Specifications
Sampling Frequency : N/A (normalized frequency)
Response           : Lowpass
Specification      : Fp,Fst,Ap,Ast
Passband Edge      : 0.09375
Stopband Edge      : 0.15625
Passband Ripple    : 0.003 dB
Stopband Atten.    : 50 dB

Measurements
Sampling Frequency : N/A (normalized frequency)
Passband Edge      : 0.09375
3-dB Point         : 0.10254
6-dB Point         : 0.10495
Stopband Edge      : 0.15625
Passband Ripple    : 0.003 dB
Stopband Atten.    : 59.2158 dB
Transition Width   : 0.0625

Implementation Cost
Number of Multipliers             : 21
Number of Adders                  : 20
Number of States                  : 10
Multiplications per Input Sample  : 21
Additions per Input Sample        : 20
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FILTER ORDER	MUL.	ADDER	STATES	MPIS	APIS
10	**21**	**20**	**10**	**21**	**20**

Fig. 3.3 cost of a Chebyshev-I Low pass filter

4. Chebyshev type II

Also known as inverse Chebyshev filters, the Type II Chebyshev filter type is less common because it does not roll off as fast as Type I, and requires more components. It has no ripple in the pass band, but does have equiripple in the stop band. The gain is:

$$G_n(\omega, \omega_o) = \frac{1}{\sqrt{1 + \frac{1}{\frac{1}{\varepsilon^2 \left({}^{\omega_0}/_{\omega}\right) T_n^2}}}} \quad \ldots\ldots\ldots\ldots \quad 4.1$$

In the stop band, the Chebyshev polynomial oscillates between -1 and 1 so that the gain will oscillate between zero and $\frac{1}{\sqrt{1+\frac{1}{\varepsilon^2}}}$ and the smallest frequency at which this maximum is attained is the cutoff frequency The ω_0 parameter ε is thus related to the stop band and attenuation γ in decibels by:

$$\varepsilon = \frac{1}{\sqrt{10^{0.1\gamma} - 1}}$$

For a stop band attenuation of 5dB, ε = 0.6801; for an attenuation of 10dB, ε = 0.3333. The $f_0 = \omega_0/2\pi$ is the cutoff frequency. The 3 dB frequency f_H is related to f_0 by:

$$f_H = \frac{f_0}{cosh\left(\frac{1}{n}\cos^{-1}\frac{1}{\varepsilon}\right)} \ldots\ldots\ldots\ldots\ldots\ldots 4.2$$

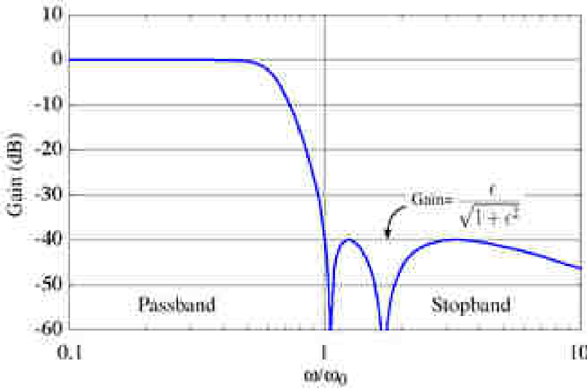

Fig.4.1 The frequency response of a 5th order type II chebyshev low pass filter with $\varepsilon = 0.01$ [6]

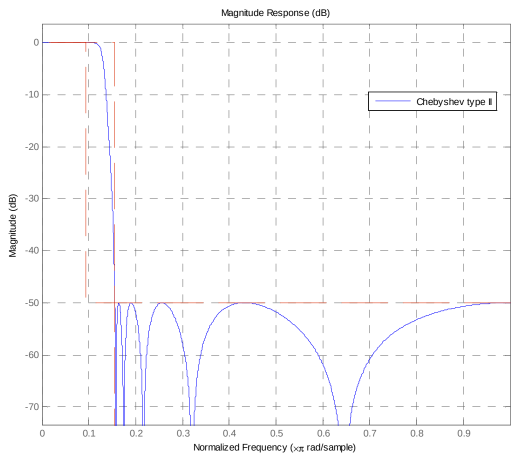

Fig. 4.2 Magnitude response of a Chebyshev-II Low pass filter with $\varepsilon = 0.009098045$

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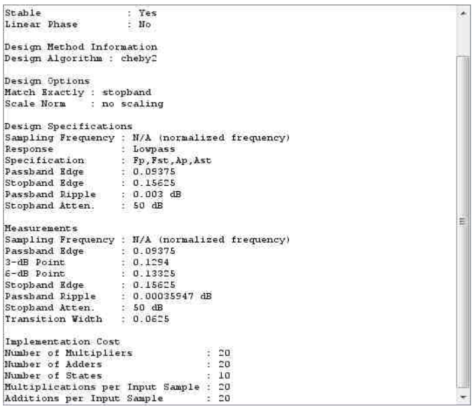

FILTER ORDER	MUL.	ADDER	STATES	MPIS	APIS
10	**20**	**20**	**10**	**20**	**20**

Fig. 4.3 cost of a Chebyshev-II Low pass filter

5. Elliptic Filter

An elliptic filter (also known as a Cauer filter, named after Wilhelm Cauer, or as a Zolotarev filter, after Yegor Zolotarev) is a signal processing filter with equalized ripple (equiripple) behavior in both the pass band and the stop band. The amount of ripple in each band is independently adjustable, and no other filter of equal order can have a faster transition in gain between the pass band and the stop band, for the given values of ripple (whether the ripple is equalized or not). Alternatively, one may give up the ability to independently adjust the pass band and stop band ripple, and instead design a filter which is maximally insensitive to component variations.

As the ripple in the stop band approaches zero, the filter becomes a type I Chebyshev filter. As the ripple in the pass band approaches zero, the filter becomes a type II Chebyshev filter and finally, as both ripple values approach zero, the filter becomes a Butterworth filter. The gain of a low pass elliptic filter as a function of angular frequency ω is given by:

$$G_n(\omega) = \frac{1}{\sqrt{1+\varepsilon^2 R_n^2(\xi, {}^{\omega}/_{\omega_0})}} \ldots\ldots\ldots\ldots\ldots\ldots .5.1$$

Where R_n is the n^{th} order elliptic rational function (sometimes known as a Chebyshev rational function) and

ω_0 is the cutoff frequency

ε is the ripple factor

ξ is the selectivity factor

The value of the ripple factor specifies the pass band ripple, while the combination of the ripple factor and the selectivity factor specify the stop band ripple.

$$G_n(\omega) = \frac{1}{\sqrt{1+\ \alpha^2({}^{1}/_{\omega})T_n^2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots .\ 5.2$$

- In the pass band, the elliptic rational function varies between zero and unity. The gain of the pass band therefore will vary between 1 and ${}^{1}/_{\sqrt{1+\varepsilon^2}}$.
- In the stop band, the elliptic rational function varies between infinity and the discrimination factor L_n which is defined as:

$L_n = R_n$=$(\ \xi, \xi)$

The gain of the stop band therefore will vary between 0 and ${}^{1}/_{\sqrt{1+\varepsilon^2 L_n^2}}$.

- In the limit of $\xi \rightarrow \infty$ the elliptic rational function becomes a Chebyshev polynomial, and therefore the filter becomes a Chebyshev type I filter, with ripple factor ε
- Since the Butterworth filter is a limiting form of the Chebyshev filter, it follows that in the limit of $\xi \rightarrow \infty$, $\omega \rightarrow 0$ and $\varepsilon \rightarrow 0$ such that $\varepsilon, R_n\ (\xi,\ {}^{1}/_{\omega_0}) = 1$ the filter becomes a Butterworth filter

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- In the limit of $\xi \to \infty$, $\varepsilon \to 0$ and $\omega \to 0$ such that $\xi\omega_0 = 1$ and $\varepsilon\, L_n = \alpha$, the filter becomes a Chebyshev type II filter with gain

$$G(\omega) = \frac{1}{\sqrt{1 + \alpha^2 (1/\omega) T_n^2}}$$

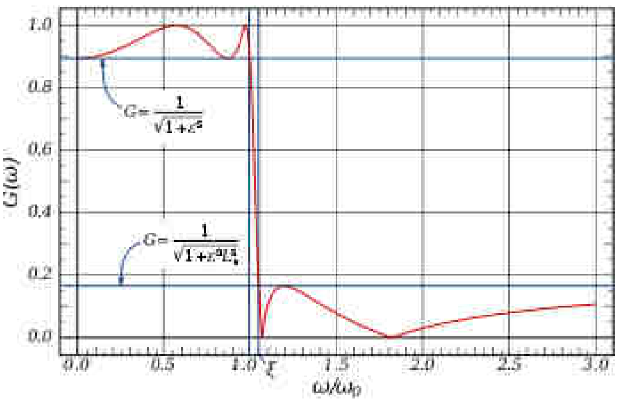


Fig. 5.5 The frequency response of a 4th order elliptic low pass filter $\varepsilon = 0.5\ and\ \xi = 1.05. A$. Also shown that the minimum gainin the pass band and the maximum gain in the stop band and the transition region between normalized frequency 1 and ξ . [6]

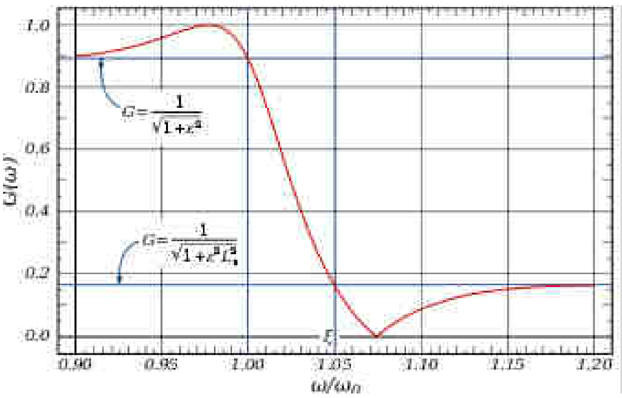


Fig.5.2. Close up the transition region of the above plot [6]

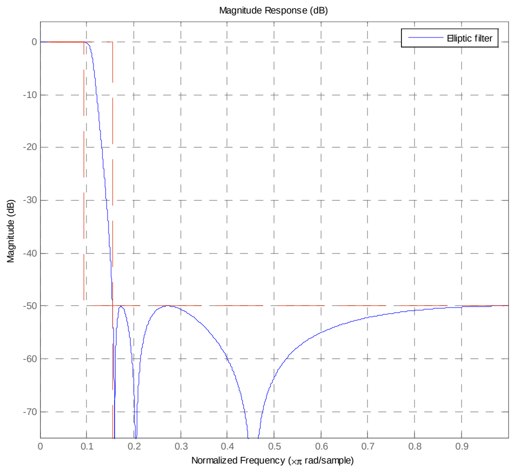
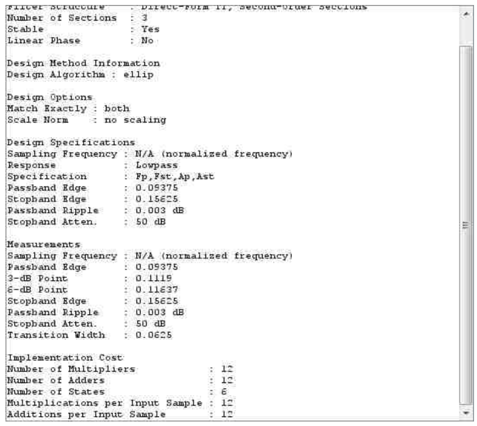


Fig. 5.3 Magnitude response of a Elliptic Low pass filter



```
Filter Structure    : Direct-Form II, Second-Order Sections
Number of Sections  : 3
Stable              : Yes
Linear Phase        : No

Design Method Information
Design Algorithm : ellip

Design Options
Match Exactly : both
Scale Norm    : no scaling

Design Specifications
Sampling Frequency : N/A (normalized frequency)
Response           : Lowpass
Specification      : Fp,Fst,Ap,Ast
Passband Edge      : 0.09375
Stopband Edge      : 0.15625
Passband Ripple    : 0.003 dB
Stopband Atten.    : 50 dB

Measurements
Sampling Frequency : N/A (normalized frequency)
Passband Edge      : 0.09375
3-dB Point         : 0.1119
6-dB Point         : 0.11637
Stopband Edge      : 0.15625
Passband Ripple    : 0.003 dB
Stopband Atten.    : 50 dB
Transition Width   : 0.0625

Implementation Cost
Number of Multipliers            : 12
Number of Adders                 : 12
Number of States                 : 6
Multiplications per Input Sample : 12
Additions per Input Sample       : 12
```

FILTER ORDER	MUL.	ADDER	STATES	MPIS	APIS
6	**12**	**12**	**6**	**12**	**12**

Fig. 5.4 cost of an Elliptic Low Pass filter

[B] PROPOSED FILTER DESIGN

A Elliptic low filter is used to shape and oversample a symbol stream before modulation/transmission as well as after modulation and demodulation. It is used to reduce the bandwidth of the oversampled symbol stream without introducing inters symbol interference.

In this proposed work Elliptic filter has been designed using filter order 06 as compare to other filters (Butterworth, Chebyshev) shown in Table 6.1.

Table 6.1
Cost analysis IIR Elliptic Low Pass filter

FILTER TYPE	ORDER	MUL.	ADDER	STATES	MPIS	APIS
Butter worth	18	36	36	18	36	36
Chebyshev type I	10	21	20	10	21	20
Chebyshev type II	10	20	20	10	20	20
Elliptic Filter	06	12	12	06	12	12

[C] RSEULTS & DISCUSSION

Parametric equalizers are designed as second-order IIR filters. These filters have the drawback that because of their low order (Elliptic low Pass filter order=6), they can present relatively large ripple or transition regions and may overlap with each other when several of them are connected in cascade.

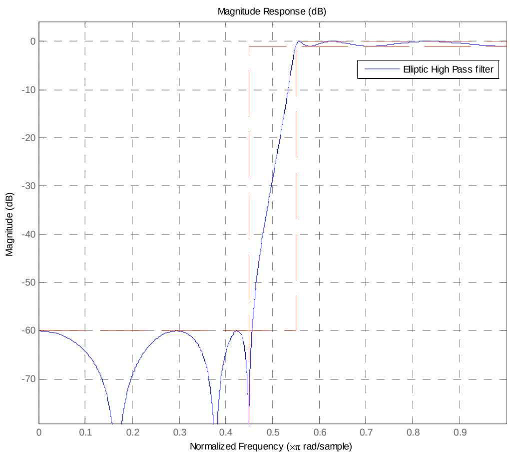


Fig. 7.1 Magnitude response of a Elliptic High Pass filter

High-order (Elliptic Pass Band Filter order = 08) designs provide much more control over the shape of each filter.

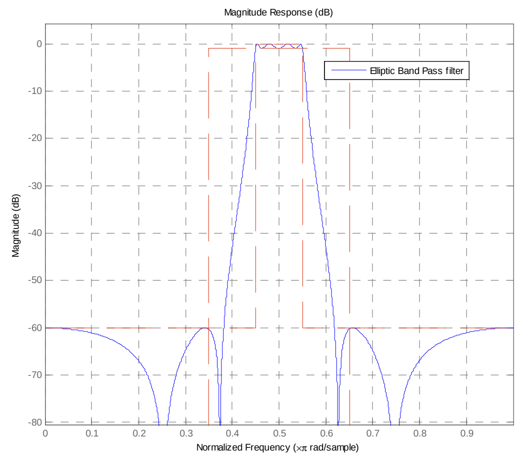


Fig. 7.2 Magnitude response of a Elliptic Band Pass filter

Notice that we have specified both a pass band gain (Gp) and a stop band gain (Gst). Given parameters (filter order = 8) allow for the Elliptic Stop band filter to ripple in the pass band and stop band with the advantage of providing steeper transitions between pass band and stop band.

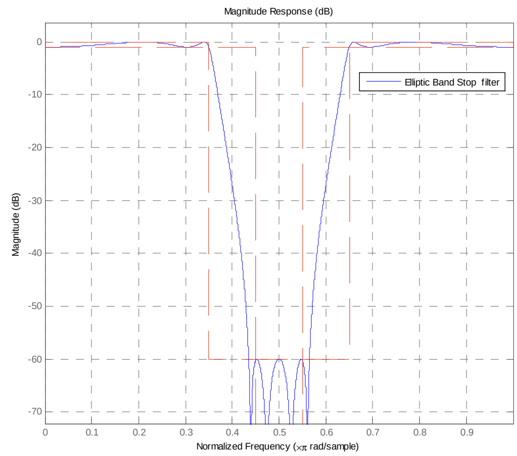


Fig. 7.3 Magnitude response of a Elliptic Band Stop filter

TABLE 7.1
IIR FILTER TYPES

FILTER TYPE	ORDER	MUL.	ADDER	STATES	MPIS	APIS
Elliptic Low Pass Filter	06	12	12	06	12	12
Elliptic High Pass Filter	06	12	12	06	12	12
Elliptic Pass						

Band Filter	08	16	16	08	16	16
Elliptic Stop Band Filter	08	16	16	08	16	16

[D] CONCLUSION

Theory and practice prove, digital audio signal Processing system using IIR digital Elliptic filter,

The Elliptical IIR Low Pass Filter is examined and comparison from the Butterworth and Chebyshev is done. The Elliptical Filter is minimizing the order of filter optimal to 75% to Butterworth Filter, and 42.5% to Chebyshev.

Elliptic Stop band filter to ripple in the pass band and stop band with the advantage of providing steeper transitions between pass band and stop band.

ACKNOWLEDGMENT

The author convey their sincere thanks to Er. Harmeet Singh (Ast. Prof. ECE), Er. Varinder Pal (Ast. Prof. ECE), Er. Harvinder Singh (Lab. Incharge, ECE).

They are also thankful Er. Amarinder Singh Bal (Ast. Prof. Electrical and Electronic Engineering) and Sanjay Kaura (Ast. Prof., Applied Physics) DIPS Institutes for value able Suggestions.

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